Factors Influencing the Psychometric Characteristics of an Adaptive Testing Strategy for Test Batteries

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A monte carlo simulation was conducted to assess the effects in an adaptive testing strategy for test batteries of varying subtest order, subtest termination criterion, and variable versus fixed entry on the psychometric properties of an existent achievement test battery. Comparisons were made among conventionally administered tests and adaptive tests using adaptive intra-subtest item selection with and without inter-subtest branching. Data consisted of responses of 300 simulees to a

201-item achievement test battery. Mean test battery length was reduced from 42.5% to 52.3% using adaptive intra-subtest item selection with variable termination. Reductions in mean subtest lengths ranged from 27% to 67%. When inter-subtest branching was added, additional test length reductions of 1% to 2% were observed for individual subtests. The reductions in test length were achieved with no significant loss of fidelity or psychometric information. The addition of inter-subtest branching resulted in levels of mean test battery information more similar to those of the full test battery, even with mean test battery reductions of 50% in number of items administered. Subtest order was shown to have no effect on the evaluative criteria employed. The results generally supported previous studies of this adaptive testing strategy. Suggestions for future research are presented.

Contents

Factors Influencing the Psychometric Characteristics of an Adaptive Testing Strategy for Test Batteries

Recent studies (e.g., Bejar, Weiss, & Gialluca, 1977; Bejar, Weiss, & Kingsbury, 1977) have demonstrated the feasibility of applying unidimensional adaptive testing strategies to measure classroom achievement. Frequently, however, achievement test batteries are composed of items drawn from several distinct content or subject matter areas. Under these circumstances, the unidimensionality assumption may be untenable or inappropriate when applied to the entire set of items. Application of a unidimensional item response theory (IRT) model to multidimensional test data usually results in achievement level estimates that reflect achievement on only a small subset of the test domain (i.e., that subset of items having loadings on the first factor). Thus, for example, although Bejar, Weiss, and Kingsbury (1977) showed that a single factor emerged from analysis of a multicontent achievement test, that factor accounted for only 23.3% of the variance of the original variables. As a consequence, unidimensional adaptive testing using that single factor would leave 76.7% of the original variance of the items unaccounted for. Similar results were reported by Reckase (1978), who showed first factors of achievement tests accounting for 1.6% to 81.4% of the variance of the original items.

Treating multicontent achievement test batteries as if they were unidimensional also results in loss of diagnostic information about a student by reducing data on a student to only one score. Frequently, however, scores on each content area are important information to be used by instructors for instructional decisions at both the individual and class level.

Brown and Weiss (1977) designed an adaptive testing strategy for use with test batteries that would reduce testing time, yet provide scores on the separate subtests in the battery. Their testing strategy included provision for adaptively branching between content area subtests as well as adaptive item selection within a content area subtest. Brown and Weiss investigated the characteristics of their combined inter-subtest/intra-subtest adaptive testing strategy using a real-data simulation of a military achievement test battery. In this approach, item response data obtained under conventional testing conditions were reanalyzed as if they were administered by the adaptive testing strategy. The findings indicated average reductions in test battery length of approximately 50% while maintaining high levels of psychometric information. Unfortunately, Brown and Weiss's (1977) results confounded the relative contributions of intra-subtest item selection and inter-subtest branching.

In an attempt to isolate the separate effects of the intra-subtest and inter-subtest components of the Brown and Weiss testing paradigm, Gialluca and Weiss (1979), also using a real-data simulation design, applied the testing strategy to a five-subtest biology test battery. Their findings showed test length reductions of 20% to 30%, again with minimal loss of psychometric information. They concluded that most of the reduction in test length was due to the adaptive intra-subtest item selection procedure and that the addition of inter-

subtest branching produced an additional reduction of 1% to 5% in test length.

Both Brown and Weiss (1977) and Gialluca and Weiss (1979) evaluated the performance of the adaptive testing strategy in similar ways. One of their evaluative criteria was correlations of achievement estimates from the adaptive tests with those from the original conventional tests. Evaluation of these correlations is difficult, however, since they are part-whole correlations that are artifically inflated due to the items administered in common by the two testing strategies. Thus, although the previous research has demonstrated essentially no loss in psychometric information due to the adaptive testing strategy, the question of the effect on score validity has not been investigated. Previous research also was restricted to the application of only one method of ordering subtests for adaptive administration: subtests in both studies were ordered by their multiple correlations with each other.

Purpose

The present study investigated the separate contributions of intra-subtest adaptive item selection and inter-subtest adaptive branching in terms of (1) reduction in the number of items administered, (2) psychometric information available in the test scores, and (3) correlations between achievement estimates derived from adaptive and conventional test administration with true achievement levels. In addition, different methods for ordering the subtests for adaptive administration were studied.

METHOD

Procedure

Test Items

Monte carlo simulation was used so that the validity question, i.e., the correlation of observed with true ability estimates, could be adequately investigated. Since a simulation study is valuable to the extent that the underlying model accurately reflects the characteristics of actual data, the achievement test data used by Brown and Weiss (1977) formed the basis for this study.

The Brown and Weiss data consisted of the response vectors of 365 Navy fire control technicians on a 232-item achievement test battery. The test battery was composed of 12 subtests, each covering a different subject matter area. The items were parameterized independently for each subtest using Urry's (1976, p. 99) Ogivia computer program utilizing the three-parameter normal ogive model.

Table 1 shows the means and standard deviations of the IRT item parameters a, b, and c from the Brown and Weiss (1977) Ogivia parameterization. (Individual item parameter estimates, by subtest, are in Appendix Table A.) Columns 2, 3, and 4 of Table 1 show that, from the total available item pool of 232 items, item parameter estimates were obtained for 87% (or 201) of these items. Subtest 12 lost the greatest number of items (28% of the original 25), whereas Subtests 1, 2, 6, and 8 did not lose any of their items.

Mean item difficulty (b) ranged from .06 for Subtest 1 to 1.44 for Subtest

Table 1
Means and Standard Deviations of Normal Ogive Item Discrimination (a),
Difficulty (b), and Pseudo-Guessing (c) Parameters
for 12 Subtests (from Brown & Weiss, 1977)

| Subtest | Number of Items | | Percent of Items Parame-terized | Parameter | | | | | |
| | Avail-able | Parame-terized | | a | | b | | c | |
				Mean	SD	Mean	SD	Mean	SD
1	10	10	100	1.90	.62	.06	1.03	.52	.11
2	10	10	100	2.12	.86	.31	1.29	.53	.18
3	18	15	83	1.80	.56	.54	1.30	.55	.08
4	22	19	86	1.60	.60	.43	1.28	.47	.08
5	18	17	94	1.57	.65	.74	1.32	.47	.10
6	18	18	100	1.58	.43	1.19	1.45	.56	.09
7	14	13	93	1.98	.94	1.20	1.26	.52	.18
8	12	12	100	2.12	.90	.84	1.10	.43	.10
9	24	22	92	1.49	.59	.88	1.36	.43	.10
10	29	23	79	1.66	.57	1.28	1.12	.44	.14
11	32	24	75	1.48	.61	.91	1.39	.43	.14
12	25	18	72	1.73	.58	1.44	1.34	.52	.17
Total	232	201	87	1.75	.54	.82	1.38	.49	.10

2; and mean item discrimination (a) ranged from 1.48 for Subtest 11 to 2.12 for Subtests 2 and 8. Mean estimates of the pseudo-guessing parameter (c) ranged from .43 for Subtests 8, 9, and 11 to .56 for Subtest 6.

Generation of Simulated Examinees

The first step in a simulation is generation of hypothetical examinees with trait levels matching known or assumed characteristics of a real population. Using the subtest score intercorrelation matrix from Brown and Weiss (1977, p. 11), 12 trait levels were generated for each examinee by the following algorithm. Twelve principal components were extracted from the subtest intercorrelation matrix. Independently distributed standard normal deviates were drawn from a random number generator and assigned, one to each component. An examinee's trait score was considered to be the sum of the product of the component loading for that trait on each of the components, multiplied by the random deviate assigned to that component. In matrix notation, this can be written as

$$\underset{\sim}{\theta} = \underset{\sim}{Z} \underset{\sim}{W} \tag{1}$$

where Z is a 1 × 12 matrix of standard normal deviates and W is a 12 × 12 matrix of factor loadings.

A 1,000 × 12 (examinee-by-subtest) matrix of generated true achievement levels was obtained for computing the subtest intercorrelation matrix to be used for ordering the subtests by multiple correlations. A random sample of 300 examinees was retained for the testing simulations. Using this examinee-by-subtest matrix of generated true achievement levels, the multivariate item response matrix was generated.

Item Response Generation

Response vectors for simulated examinees were generated based on the item parameters for items measuring several latent traits. Given θ_{ik} (simulee i's true ability on trait k) and the parameters for item g on trait k, the probability of simulee i responding correctly was calculated as

$$P_{gk}(\theta_{ik}) = c_{gk} + (1 - c_{gk}) \, \Psi \, [Da_{gk}(\theta_{ik} - b_{gk})] \tag{2}$$

where

a_{gk} is the discrimination for item g measuring trait k,

b_{gk} is the difficulty of item g on trait k,

c_{gk} is the pseudo-guessing level of item g on trait k,

D is 1.7, a scaling constant for the logistic model,

and $\Psi[x]$ is the logistic cumulative distribution function.

The probabilities thus computed were compared to a vector of uniformly distributed random numbers, r_u, in the interval 0 to 1. If

$$r_u \leq P_{gk}(\theta_{ik}), \quad u_{gik} = 1 \tag{3}$$

or if

$$r_u > P_{gk}(\theta_{ik}), \quad u_{gik} = 0. \tag{4}$$

Conventional Test

To provide a basis for comparison of the adaptive testing strategy, conventional administration of the achievement battery was also simulated. The subtests were administered sequentially, with all items within a subtest given in order and all examinees taking all the items in the same order. Owen's (1975) Bayesian scoring algorithm was used to score each of the subtests, with a mean of 0.0 and variance of 1.0 as the initial prior estimate for each subtest. That is, no differential item selection or differential subtest order occurred for the conventional test.

Adaptive Testing Strategy

As in the Brown and Weiss (1977) and Gialluca and Weiss (1979) studies, an adaptive testing strategy utilizing both intra-subtest item selection based upon the item information function and a regression-based inter-subtest branching rule were used. Three ways of ordering the subtests, for use with the regression approach to estimate differential subtest entry, were simulated: (1) ordering on the basis of highest multiple correlation (as in the two previous studies), (2) ordering on the basis of number of items in the subtest, and (3) random ordering.

Additionally, an adaptive testing strategy employing only intra-subtest item selection was simulated. This was done in order to separate the effects of variable termination in the intra-subtest item selection strategy from those of the inter-subtest branching strategy. Instead of differential entry into subsequent subtests based on information from previous subtests, each subtest was

treated individually, as in the conventional test. As with the conventional test, Bayesian scoring was used, with a mean of 0.0 and variance of 1.0 as the initial prior $\hat\theta$ for each of the subtests. Hence, the only difference between these tests and the other adaptive tests was that inter-subtest branching (and, therefore, a differential Bayesian prior achievement estimate for each simulee on each subtest) was not used.

Intra-Subtest Adaptive Testing

Item selection. Adaptive intra-subtest item selection used a maximum information item selection method in which the item to be administered to each simulee at each stage of the test was that item which provided the most information at that simulee's current achievement level estimate (see Brown & Weiss, 1977, for a detailed explanation of this method). Hence, for each simulee, the item information value for each unadministered item within a subtest was evaluated from

$$I(\hat\theta) = \frac{[P'_g(\hat\theta)]^2}{P_g(\hat\theta)[1 - P_g(\hat\theta)]} \qquad [5]$$

where $P_g(\hat\theta)$ is defined as in Equation 2 and P' is the first derivative of the item characteristic curve evaluated at $\hat\theta$, the simulee's current achievement level estimate. The item that had the highest information value was selected for administration; once an item was administered to a simulee, it was omitted from further consideration for that simulee.

Scoring and termination. As in the conventional test, Bayesian achievement level estimates were obtained for each simulee after administration of each item. Termination of testing within a subtest was governed by two criteria: (1) testing terminated when all items within a subtest had been administered or (2) when no item remaining in a subtest provided a predetermined amount of information at the current estimate of θ. Two minimum values of information were used in this study: .01 and .05.

Inter-Subtest Adaptive Branching

Subtest ordering. Three methods of ordering the subtests were simulated. In the first method, adapted from Brown and Weiss (1977), subtests were ordered on the basis of their highest multiple correlation with each other. Brown and Weiss, however, ordered subtests based on linear regression of number-correct scores. In this study, the simulee-by-subtest matrix of generated true achievement levels (N = 1,000) was intercorrelated and the resulting correlation matrix was used as the basis for inter-subtest branching. The highest zero-order correlation was chosen from this intercorrelation matrix, one of the two subtests was arbitrarily chosen to be administered first, and the other was administered second. Multiple correlations were then computed using the subtests previously chosen first and second as predictors, with each of the remaining subtests, in turn, being designated as the criterion. The subtest having the highest correlation with the predictor subtests was chosen to be administered next. By adding one subtest to the predictor set at each succeeding stage, all subtests were thus ordered.

Ordering subtests on the basis of this stepwise regression procedure requires [N(N-1)/2]-N regression equations, whereas given an a priori subtest ordering, only N-1 regression equations are required to estimate differential individual entry achievement estimates. Although the ordering of subtests by stepwise linear regression appears to be a natural adjunct, two other procedures were implemented so that the stepwise procedure could be evaluated against alternatives.

One alternative, suggested by Brown and Weiss (1977), was to order subtests based on the number of items. It seems logical to administer the longer subtests first, since at the early stages of testing a wider range of items would enable a more accurate assessment of achievement levels. As the differential entry achievement estimates became more accurate, fewer items would be required to assess the examinee's achievement levels on subsequent tests.

Subtest order may have an effect on the psychometric characteristics of the adaptive testing strategy or on number of items administered. If it does not, then it should be possible to obtain the same results by ordering subtests randomly. To test this hypothesis, subtests were ordered randomly as a third way of ordering them for adaptive administration.

Differential subtest entry. After administration of the first subtest, entry achievement estimates for subsequent subtests were differentially determined for each simulee. For the first subtest, however, each simulee's entry achievement estimate was determined by setting its prior achievement level estimate to be $\hat{\theta} = 0.0$ and selecting that item from the initial subtest that provided the most information at $\hat{\theta} = 0.0$. Thus, all simulees began the initial subtest with the same item.

Entry into the item pool for the next subtest was determined by the bivariate regression of scores from that subtest on the first subtest and the simulee's estimated achievement level on the first subtest. This yielded an estimate of the simulee's achievement level on the second subtest. This achievement level estimate then became the Bayesian prior for intra-subtest item selection for the second subtest. The squared standard error of estimate from the bivariate regression equation became the Bayesian prior variance.

In general, an examinee's final achievement level estimates from all n previously administered subtests were used in the appropriate linear multiple regression equation for predicting the n + 1st subtest score. The squared standard error of estimate from each regression was used as the Bayesian prior variance for that subtest.

Dependent Variables

Validity and Test Length

In a simulation study, true or generated ability or achievement levels establish the criterion. Thus, correlations of achievement level estimates on each subtest with their known true values were obtained for all testing conditions. In addition, the number of items administered under each testing condition was examined for each subtest.

Test Information

The precision of measurement at different achievement levels is indexed by its information value, a quantity that is inversely proportional to the squared length of the confidence interval about an achievement level estimate (Hambleton & Cook, 1977). Subtest information curves were computed by evaluating the item information function at each simulee's estimated achievement level and summing over all items administered in that subtest. Examinees were then grouped into equally spaced nonoverlapping intervals, and the mean information over all simulees within an interval was plotted at the midpoint of that interval to obtain the subtest information curves.

Statistics descriptive of information curves. Although graphs are a simple and convenient way to present information curves, they are difficult to interpret directly when many curves are involved. Thus, for comparison purposes, the means and coefficients of variation of the information curves were computed.

Mean or average information is a statistic that is not disproportionately weighted by the distributional characteristics of the examinee group tested. The greater the mean information, the more precise are the achievement level estimates, on the average, over all levels of achievement. The coefficient of variation (Snedecor & Cochran, 1967, p. 62), which is equal to the standard deviation of the information curve divided by its mean, is of interest because its departure from zero indicates that the goal of equiprecise measurement is not being met.

Relative information. Relative information may be defined as

$$RI_{A/B} = I_A(\theta)/I_B(\theta)$$

[6]

where $I_A(\hat{\theta})$ is the information value of test A and $I_B(\hat{\theta})$ is the information value of test B (Bejar, 1977). A useful interpretation is in terms of test length (Lord, 1980): when $RI_{A/B} > 1.00$ at a given achievement level, test A provides an amount of information equal to test B lengthened by a factor of $RI_{A/B}$. In the present study, for example, relative information was computed for each adaptive subtest relative to its conventional counterpart. That is, the numerator was the value of information for the adaptive subtest and the denominator was the value of information for that same subtest administered conventionally.

RESULTS

Subtest Order

The intercorrelations of the simulee-by-subtest matrix of generated true achievement levels are shown in the lower triangle of Table 2. The residuals from the target correlation matrix (Brown & Weiss, 1977, p. 11) are shown in the upper triangle of Table 2. The largest absolute residual was .08. The average absolute deviation from the target matrix was .03, so that, overall, the target matrix appeared to be reproduced faithfully. However, the reconstructed correlation matrix was sufficiently different to change the rank ordering of the subtests based on the highest multiple correlations with each other, from the rank orderings obtained by Brown and Weiss (1977). The highest bivariate correlation

(.52) was observed between Subtests 3 and 11, which were designated to be administered first and second, respectively.

Table 2
Intercorrelations Among True Achievement Levels (Lower Triangle)
and Residuals from Generated Correlation Matrix (Upper Triangle)

Subtest	Subtest											
	1	2	3	4	5	6	7	8	9	10	11	12
1		-02	00	01	-04	-05	-01	-04	-01	-03	-01	-02
2	29		-07	-05	00	-04	-01	-01	00	01	-05	06
3	40	30		-04	-01	-05	00	-02	-04	-02	-01	-01
4	37	35	42		-02	-04	-03	-05	-06	-06	-02	-01
5	34	37	47	36		-05	00	-03	00	00	-03	00
6	25	22	31	35	33		-01	-06	-01	-04	-08	-01
7	29	37	41	33	46	34		-03	02	-01	02	-01
8	21	28	27	23	32	24	33		-02	-02	-04	01
9	22	33	38	42	47	44	43	26		-05	-01	-04
10	16	36	25	27	28	29	32	25	35		-04	01
11	41	28	52	37	38	22	39	24	34	23		06
12	25	33	21	13	29	15	26	27	22	32	32	

Note. Decimal points omitted.

Multiple regression equations were obtained by using Subtests 3 and 11 as predictor variables and each of the remaining subtests, in turn, as criterion variables. This procedure continued until all the subtests had been ordered. Table 3 contains the multiple correlations for each subtest predicted from all previous subtests and the subsequent ordering of the subtests based on the multiple correlations. The raw score regression weights, regression constants, and squared standard errors of estimate are shown in Appendix Table B. Table 4 shows the ordering sequence of each subtest based on the following three order-

Table 3
Multiple Correlations Among Ordered Subtests

Criterion Subtest	Predictor Subtest										
	3	11	5	7	9	4	1	6	2	10	12
11	52										
5	47	50									
7	41	46	54								
9	38	41	51	55							
4	42	45	48	49	53						
1	40	46	48	48	49	57					
6	31	32	38	41	49	51	51				
2	30	33	41	45	46	48	49	49			
10	25	27	32	36	41	42	42	42	47		
12	21	32	37	38	38	38	40	40	44	47	
8	27	29	36	39	40	40	40	41	42	43	44

Note. Decimal points omitted.

ing methoding (1) highest multiple correlation, (2) number of items, and (3) random. Regression weights and related data for the latter two ordering methods are in Appendix Tables C and D.

Table 4
Order of Administration of
12 Subtests Resulting from
Three Ordering Methods

Ordering Method		
Highest R	Number of Items	Random
3	11	3
11	10	4
5	9	11
7	4	2
9	12	7
4	6	6
1	5	1
6	3	5
2	7	8
10	8	10
12	2	12
8	1	9

Test Length

Summary statistics for the number of items administered in each of the 12 subtests for each subtest order and inter-subtest branching condition, and two levels of intra-subtest termination are given in Tables 5 and 6.

Adaptive intra-subtest item selection. The data in Table 5 summarize the reductions in mean test length observed when subtests were administered adaptively but with no inter-subtest branching. That is, each subtest was administered as an independent subtest using only adaptive item selection within each subtest. Thus, each examinee began each subtest with a Bayesian prior achievement estimate of 0.0 and a prior variance of 1.0. The length of the total test battery averaged 95.78 items under the termination criterion of .05 and 115.55 items under the .01 termination criterion.

The maximum number of items administered for the total battery using only adaptive intra-subtest item selection with .05 termination was 154, which was a 23% reduction in total test length; for the .01 termination condition, the maximum number of items administered was 171, which was a 15% reduction in total test battery length. The shortest adaptive battery yielded a 78% reduction in test length for .05 termination (44 items) and a 72% reduction for .01 termination (56 items). The average reduction in test length over the total test battery (i.e., the average of the subtest mean reductions weighted by the number of items in each subtest) was 52.3% for the .05 termination and 42.5% for the .01 termination.

Table 5
Number of Items Administered in 12 Adaptive and Conventional Subtests
with Intra-Subtest Item Selection Only at Two Termination Levels

Sub-test	Conven-tional Test	Termination Criterion of .05					Termination Criterion of .01				
		Mean	SD	Range Min	Max	Percent Reduc-tion	Mean	SD	Range Min	Max	Percent Reduc-tion
1	10	6.07	1.23	3	8	39.3	7.29	.87	5	8	27.1
2	10	6.12	1.00	3	8	38.8	7.00	.80	4	8	30.0
3	15	7.61	1.42	3	11	49.3	9.12	1.50	4	12	39.2
4	19	10.93	2.20	6	15	42.5	12.99	1.89	6	15	31.6
5	17	9.12	2.30	5	12	46.4	10.98	2.60	6	14	35.4
6	18	7.55	1.99	3	10	58.0	8.78	2.03	4	15	51.2
7	13	5.19	1.60	2	12	60.1	5.63	1.42	2	12	56.7
8	12	5.88	2.05	3	10	51.0	7.16	2.14	3	11	40.4
9	22	11.73	2.68	6	17	46.7	14.09	2.90	8	19	35.9
10	23	7.54	4.14	3	21	67.2	10.78	4.58	3	21	53.1
11	24	12.09	2.58	5	17	49.6	14.52	1.82	8	20	39.5
12	18	5.95	1.99	2	13	66.9	7.21	2.14	3	16	60.0
Total Battery	201	95.78		44	154	52.3	115.55		56	171	42.5

*Computed by the formula 100-[(mean number of items in adaptive test/number of items in conventional test) × 100]

The largest average reduction in subtest length using a termination crite-rion of .05 occurred for Subtest 10 and amounted to an average decrease of 67% of the items. The smallest average decrease was observed for Subtest 2, which showed a 39% reduction. For the .01 termination rule the largest reduction oc-curred for Subtest 12 with an average 60% reduction, while Subtest 1 showed the smallest average reduction of 27%.

Inter-subtest branching. When the inter-subtest branching strategy was employed to implement differential subtest entry in addition to adaptive intra-subtest item selection, test lengths were generally reduced even further. Table 6 shows the mean test lengths when subtests were ordered by multiple correla-tion, by the number of items, and randomly. However, compared to the results from intra-subtest item selection alone (Table 5), these reductions were slight.

For example, Table 5 shows that the mean total test battery length when using only intra-subtest adaptive item selection with a termination criterion of .05 was 95.78, or a 52.3% reduction. When coupled with inter-subtest adaptive branching (Table 6), mean test battery lengths decreased to 94.10 (53.2% reduc-tion) for ordering by multiple correlation, 93.67 (53.4% reduction) for ordering by number of items, and 94.94 (52.8% reduction) for random ordering. For the .01 termination criterion, intra-subtest item selection alone yielded a mean test battery length of 115.55, or a 42.5% reduction. When inter-subtest branch-ing was added, mean test battery lengths of 115.29, 114.45, and 114.57 were ob-served, representing mean test battery reductions of 42.6%, 43.0%, and 43.0% from the full test battery length. In general (across all three ordering proce-dures), additional reductions in test length due to inter-subtest branching

Table 6
Number of Items Administered in 12 Adaptive Subtests Ordered
by Highest Multiple Correlation, by Number of Items, and Randomly,
Using Differential Subtest Entry

Ordering Method and Subtest	Conventional Test	Termination Criterion of .05		Range		Percent Reduction	Termination Criterion of .01		Range		Percent Reduction
		Mean	SD	Min	Max	tion	Mean	SD	Min	Max	tion
Highest Multiple Correlation											
3	15	7.55	1.43	3	11	49. 7	9.09	1.50	4	12	39.4
11	24	11.83	2.77	4	15	50. 7	14.42	2.12	7	19	39.9
5	17	9.09	2.26	3	12	46. 5	11.14	2.44	5	14	34.5
7	13	4.90	1.38	1	11	62. 3	5.74	1.61	1	12	55.8
9	22	11.54	2.92	3	17	47. 5	13.96	3.11	5	18	36.5
4	19	10.75	2.14	4	14	43. 4	12.88	1.81	5	15	32.2
1	10	6.16	1.24	1	8	38. 4	7.52	.93	3	9	24.8
6	18	7.13	1.85	2	10	60. 4	8.68	2.09	2	14	51.8
2	10	5.87	1.24	2	8	41. 3	6.76	1.00	3	8	32.4
10	23	7.94	4.09	2	21	65. 5	10.77	4.84	2	21	53.2
12	18	5.86	1.69	2	12	67. 4	7.18	2.20	2	16	60.1
8	12	5.48	2.13	1	10	54. 3	7.15	2.23	2	11	40.4
Total	201	94.10		28	149	53. 2	115.29		41	169	42.6
Number of Items											
11	24	11.93	2.62	6	16	50. 3	14.50	2.01	8	20	39.6
10	23	7.81	3.92	2	20	66. 0	10.48	4.69	2	21	54.4
9	22	11.51	2.90	5	16	47. 7	13.99	3.07	7	19	36.4
4	19	10.64	2.23	4	14	44. 0	12.83	2.04	5	15	32.5
12	18	5.93	1.92	2	15	67. 0	7.23	2.25	2	15	59.8
6	18	7.21	1.91	2	10	59. 9	8.80	1.98	3	14	51.1
5	17	9.03	2.32	3	13	46. 9	10.82	2.65	5	14	36.4
3	15	7.36	1.39	2	11	50. 9	8.99	1.70	3	11	40.1
7	13	4.64	1.54	1	12	64. 3	5.64	1.77	1	12	56.6
8	12	5.60	2.22	1	9	53. 3	7.02	2.20	2	10	41.5
2	10	5.90	1.25	2	9	41. 0	6.69	1.01	2	8	33.1
1	10	6.11	1.34	1	8	38. 9	7.46	.81	5	9	25.4
Total	201	93.67		31	153	53. 4	114.45		45	168	43.0
Random											
3	15	7.53	1.55	3	11	49. 8	9.09	1.50	4	12	39.4
4	19	10.89	2.08	5	15	42. 7	12.89	1.82	5	15	32.1
11	24	11.92	2.81	3	16	50. 3	14.35	2.08	5	20	40.2
2	10	5.91	1.25	2	8	40. 9	6.73	1.02	3	8	32.7
7	13	4.92	1.60	1	12	62. 2	5.57	1.72	1	12	57.2
6	18	7.36	1.74	2	10	59. 1	8.83	2.01	3	15	50.9
1	10	6.04	1.36	1	8	39. 6	7.48	.90	4	10	25.2
5	17	8.89	2.34	2	12	47. 7	10.87	2.72	3	15	36.1
8	12	5.72	2.12	1	10	52. 3	7.13	2.26	2	10	40.6
10	23	8.04	4.27	2	21	65. 0	10.51	4.63	2	22	54.3
12	18	6.05	1.81	1	12	66. 4	7.10	2.10	3	14	60.6
9	22	11.67	3.70	4	17	46. 9	14.02	3.22	6	18	36.3
Total	201	94.94		27	152	52. 8	114.57		41	171	43.0

*Computed by the formula 100-[(mean number of items in adaptive test/number of items in conventional test) × 100]

amounted to less than 1% of original test length for the .05 termination crite-
rion and virtually no improvement for the .01 termination criterion.

Analysis of the percentage reduction data for individual subtests due to
the various order-by-termination combinations revealed nearly identical results
across ordering methods within each termination criteria. The only discernible
trend was for the more stringent termination criterion to administer approxi-
mately 20% more items per subtest.

Correlation with True Achievement Levels

Table 7 presents the values of the correlation coefficients of the Bayesian
estimated achievement levels with the true achievement levels for the conven-
tional and adaptive tests. Generally, these correlations were fairly consistent
across both termination criteria and branching strategies.

Table 7

Correlations of Bayesian Achievement Level Estimates with the True
Achievement Levels for 12 Subtests Using Conventional Administration,
Adaptive Intra-Subtest Item Selection Only, and Intra-Subtest Combined
with Three Kinds of Inter-Subtest Ordering, at Two Termination Levels

		Termination Criterion of .05				Termination Criterion of .01			
	Conven-	Intra-	Inter-Subtest Ordering			Intra-	Inter-Subtest Ordering		
	tional	Subtest	Multiple	No.		Subtest	Multiple	No.	
Subtest	Test	Only	R	Items	Random	Only	R	Items	Random
1	.82	.81	.81	.80	.83	.80	.78	.82	.82
2	.81	.78	.77	.82	.79	.83	.79	.80	.81
3	.83	.77	.79	.83	.80	.79	.79	.79	.79
4	.84	.79	.84	.84	.82	.85	.83	.81	.84
5	.85	.80	.82	.83	.83	.85	.85	.82	.82
6	.79	.76	.79	.81	.78	.84	.79	.80	.79
7	.73	.66	.64	.73	.71	.70	.64	.67	.67
8	.82	.79	.79	.82	.80	.84	.82	.78	.79
9	.84	.87	.85	.87	.86	.86	.85	.86	.87
10	.79	.73	.77	.78	.77	.81	.76	.79	.75
11	.89	.88	.88	.87	.89	.89	.90	.89	.89
12	.72	.63	.71	.75	.68	.71	.67	.67	.67
Median	.82	.79	.79	.82	.82	.83	.79	.80	.79

Adaptive intra-subtest item selection. For the termination criterion of
.05 with no inter-subtest branching, the correlations ranged from a low of .63
for Subtest 12 to a high of .88 for Subtest 11. Using a termination criterion
of .01, the correlations ranged from a low of .70 for Subtest 7 to a high of .89
for Subtest 11. The more stringent termination criterion resulted in somewhat
higher correlations and reduced the range of the observed correlations.

Inter-subtest branching. When inter-subtest branching was added to the
adaptive intra-subtest item selection, the observed correlations showed no ap-
parent trends with respect to termination criterion or ordering procedure.

Although somewhat lower fidelity coefficients were noted for adaptive versus conventional administration, it should be remembered that, on the average, the adaptively administered subtests used 50% fewer items to estimate the achievement levels of each simulee than did the conventionally administered subtests. Yet for many subtests, differences in these fidelity correlations were quite small; in some cases the fidelity coefficients were even slightly higher for the adaptive tests.

Information

Mean information values are shown in Table 8 for Subtest 8 [$I(\hat{\theta})$] at intervals of estimated achievement ($\hat{\theta}$) for the conventional tests, and for the adaptive tests with test termination criteria of .05 and .01 using both intra-subtest item selection only and inter-subtest branching, with subtests ordered by highest multiple correlations (Subtest 8 was the last test administered under the highest multiple R ordering). These values are based on the mean informa-

Table 8
Mean Information Values [$I(\hat{\theta})$] and Number of Simulees (N) at Intervals
of Estimated Achievement Levels ($\hat{\theta}$) for Subtest 8 Using Termination
Criteria of .05 and .01 with Intra-Subtest Item Selection Only
and Combined with Inter-Subtest Branching

$\hat{\theta}$ Range		Conventional Test		Adaptive Intra-Subtest Item Selection: Termination Criterion				Adaptive Intra-Subtest Item Selection with Inter-Subtest Branching: Termination Criterion			
				.05		.01		.05		.01	
Lo	Hi	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$
-2.00	-1.80	–	–	–	–	–	–	–	–	–	–
-1.79	-1.60	–	–	–	–	–	–	1	.02	3	.02
-1.59	-1.40	6	.09	6	.08	5	.08	6	.06	8	.12
-1.39	-1.20	29	.73	19	.65	18	.54	7	.66	11	.64
-1.19	-1.00	14	2.06	5	2.23	7	2.21	16	2.29	11	2.07
-.99	-.80	–	–	1	3.82	4	3.92	1	3.88	4	4.10
-.79	-.60	22	3.38	–	–	–	–	14	3.33	11	3.28
-.59	-.40	24	2.30	49	2.18	48	2.44	52	1.98	33	2.08
-.39	-.20	36	1.54	49	1.48	37	1.54	30	1.46	37	1.54
-.19	.00	41	1.76	20	1.67	38	1.72	33	1.66	39	1.77
.01	.20	22	2.35	41	2.18	31	2.26	31	2.29	29	2.26
.21	.40	24	3.43	25	2.98	26	3.16	29	3.24	25	3.25
.41	.60	19	4.52	30	4.36	24	4.42	20	4.48	28	4.46
.61	.80	12	4.97	12	4.92	9	4.96	16	4.91	11	4.96
.81	1.00	8	4.70	4	4.83	16	4.76	10	4.75	12	4.75
1.01	1.20	14	4.65	10	4.74	18	4.75	9	4.69	14	4.65
1.21	1.40	6	5.25	2	4.92	3	4.92	6	5.10	9	5.10
1.41	1.60	15	5.31	6	5.33	4	5.34	5	5.26	5	5.18
1.61	1.80	2	4.55	10	4.68	7	4.68	4	4.82	2	4.32
1.81	2.00	8	2.96	11	2.11	5	3.21	10	2.61	8	2.57
Total Group		300	2.75	300	2.58	300	2.77	300	2.73	300	2.76

tion in the test items actually administered to each simulee, using its $\hat{\theta}$ at the termination of each subtest. The differences in mean information between the two termination conditions tended to be small, and no trends were apparent. The strong similarities in the data for individual subtests resulting from the two termination criteria suggested that little was to be gained by use of the more stringent .01 termination criterion. Thus, the remaining analyses were conducted using only the .05 criterion. Appendix Tables E through P present the mean values of estimated information at intervals of $\hat{\theta}$ for the conventional and adaptive subtests under all test conditions for the termination criterion of .05.

Adaptive intra-subtest item selection. Based on the data in Appendix Tables E through P for the conventional test and the adaptive test with adaptive inter-subtest item selection only, Table 9 presents the means of the average subtest information values across levels of $\hat{\theta}$, their coefficients of variation, and their efficiencies relative to the conventional test. The mean values of subtest information for each subtest were virtually identical for the adaptive test compared to the conventional test. The coefficients of variation for the adaptive subtests also were quite similar to the full length conventional subtests. The efficiency of adaptive intra-subtest item selection relative to the full length conventional subtests appear in the last column of Table 9. These data suggest that there was little, if any, loss of information incurred by the adaptive strategy, even though the adaptive subtests averaged 50% fewer items.

Table 9

Mean of Average Test Information, Coefficient of Variation (CV),
Range of Mean Test Information Across $\hat{\theta}$ Levels, and
Relative Mean Efficiency to Conventional Test for Adaptive Tests with
Intra-Subtest Item Selection Only and Termination Criterion of .05

Subtest	Conventional Test		Adaptive Test				Relative Mean Efficiency
	Mean	CV	Mean	CV	Range Min	Max	
1	2.75	.57	2.73	.64	.25	6.43	.99
2	2.35	.22	2.46	.35	1.66	6.78	1.05
3	2.72	.29	2.76	.30	.46	4.30	1.01
4	3.25	.14	3.17	.16	.80	5.10	.98
5	3.22	.26	3.09	.29	.65	4.31	.96
6	2.17	.18	2.08	.24	.24	2.62	.96
7	1.25	.61	1.19	.56	.44	5.03	.95
8	2.75	.53	2.58	.52	.08	5.34	.94
9	4.26	.22	4.23	.24	.72	6.86	.99
10	2.46	.51	2.40	.52	.34	12.04	.98
11	5.50	.43	5.55	.44	.54	8.61	1.01
12	1.75	.48	1.81	.44	.05	6.52	1.03
Total Battery	34.43		34.05				.99

Figure 1 graphically depicts the overall test information curves obtained when the tests were administered conventionally, with only adaptive intra-subtest item selection, and with inter-subtest branching in addition to intra-subtest item selection. Whereas Table 9 showed average values of information over

achievement levels, Figure 1 shows the mean of the average information values over subtests as a function of achievement level. The curves are very similar, with the largest differences occurring at the $\hat{\theta}$ interval -1.59 to -1.40, where mean information of the adaptive intra-subtest strategy was 14.66 and that of the conventional test was 7.82; and at the $\hat{\theta}$ intervals .61 to .80, .81 to 1.00, 1.01 to 1.20, and 1.41 to 1.60 where the conventional test's information values were 37.61, 32.01, 29.10, and 26.54, respectively, and those of the adaptive test using only adaptive intra-subtest item selection were 30.13, 25.51, 22.10, and 20.04, respectively.

Figure 1
Mean Test Information Curves Across 12 Subtests for Conventional Test
and for Adaptive Test Using Intra-Subtest Item Selection Only
and Combined with Inter-Subtest Branching

Inter-subtest branching. Table 10 presents the mean subtest information values, their coefficients of variation, and efficiencies relative to the conventionally administered subtests for the adaptive tests using three methods of ordering subtests for inter-subtest branching. Comparison of the data in Table 10 with that of Table 9 shows that the addition of inter-subtest branching to the adaptive intra-subtest item selection appeared to have a minimal effect on these evaluative criteria. Compared to the conventional test, the adaptive test with inter-subtest branching showed the same pattern of overall subtest mean information, as reflected in relative mean efficiencies very close to 1.00 for all subtests and branching conditions.

Figure 1 also shows the average information curve across the 12 subtests for the inter-subtest branching strategy using highest multiple correlations.
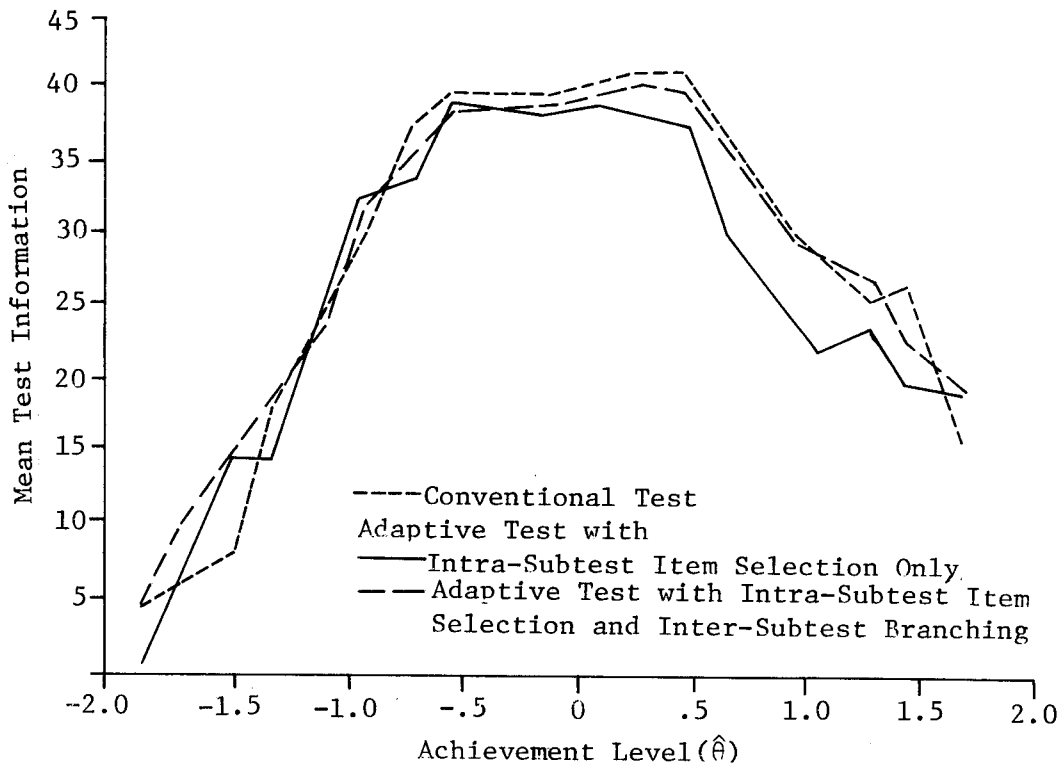
Table 10
Mean of Average Test Information, Coefficient of Variation (CV),
Range of Mean Test Information Across $\hat{\theta}$ Levels, and Relative Mean
Efficiency to Conventional Test for Adaptive Tests with Three Types
of Inter-Subtest Branching and Termination Criterion of .05

Ordering Method and Subtest	Conventional Test		Adaptive Test				Relative Mean Efficiency
					Range		
	Mean	CV	Mean	CV	Min	Max	
Highest Multiple Correlation							
3	2.72	.29	2.76	.30	.46	4.34	1.01
11	5.50	.43	5.29	.46	.40	8.61	.96
5	3.22	.26	3.08	.29	.43	4.28	.96
7	1.25	.61	1.31	.68	.12	9.51	1.05
9	4.26	.22	4.33	.22	.21	6.58	1.02
4	3.25	.14	3.15	.16	.44	7.07	.97
1	2.75	.57	2.71	.58	.13	6.46	.98
6	2.17	.18	2.11	.23	.04	2.63	.97
2	2.35	.22	2.44	.32	1.13	6.78	1.04
10	2.46	.51	2.53	.61	.14	12.16	1.03
12	1.75	.48	1.84	.47	.10	7.86	1.05
8	2.75	.53	2.73	.49	.02	5.36	.99
Total Battery	34.43		34.28				.99
Number of Items							
11	5.50	.43	5.28	.45	.81	8.60	.96
10	2.46	.51	2.33	.49	.09	8.34	.95
9	4.26	.22	4.24	.25	.59	6.96	.99
4	3.25	.14	3.16	.17	.63	7.35	.97
12	1.75	.48	1.83	.53	.04	8.17	1.04
6	2.17	.18	2.11	.21	.13	2.63	.97
5	3.22	.26	3.05	.30	.50	4.33	.95
3	2.72	.29	2.72	.28	.14	4.29	1.00
7	1.25	.61	1.28	.78	.11	14.52	1.02
8	2.75	.53	2.76	.51	.02	5.30	1.00
2	2.35	.22	2.40	.27	1.14	6.66	1.02
1	2.75	.57	2.73	.61	.08	6.44	.99
Total Battery	34.43		33.89				.98
Random							
3	2.72	.29	2.76	.32	.46	4.34	1.01
4	3.25	.14	3.17	.12	.86	5.54	.98
11	5.50	.43	5.55	.43	.15	8.61	1.01
2	7.35	.22	2.41	.31	1.14	6.77	1.02
7	1.25	.61	1.41	1.12	.06	14.60	1.13
6	2.17	.18	2.15	.20	.08	2.64	.99
1	2.75	.57	2.71	.59	.11	6.41	.98
5	3.22	.26	3.03	.28	.35	4.25	.94
8	2.75	.53	2.70	.54	.02	5.36	.98
10	2.46	.51	2.42	.55	.27	11.97	.98
12	1.75	.48	1.85	.45	.03	6.30	1.06
9	4.26	.22	4.32	.24	.36	5.67	1.01
Total Battery	34.43		34.48				1.00

Here, again, there is very little separation between the conventional test's curve and the curve for inter-subtest branching. One effect of intra-subtest item selection combined with inter-subtest branching over intra-subtest item selection alone was to narrow the gap between the conventional test and the adaptive test in the $\hat{\theta}$ intervals where intra-subtest item selection alone fell below the conventional test. Similar results were obtained when subtests were ordered by number of items and randomly for use in the adaptive inter-subtest branching procedure.

DISCUSSION

This study has replicated previous findings (Brown & Weiss, 1977; Gialluca & Weiss, 1979) that showed that an adaptive testing strategy combining intra-subtest adaptive item selection with inter-subtest branching could significantly reduce the length of an achievement test battery while maintaining desirable psychometric properties. The present study applied this test design to the same basic data set used by Brown and Weiss (1977) but used a monte carlo simulation to assess the separate effects of adaptive intra-subtest item selection and inter-subtest branching on test length, test information, and score fidelity. In addition, this study investigated the effects of various subtest orderings for inter-subtest branching on the psychometric properties of the subtests.

Adaptive Intra-Subtest Item Selection

The adaptive intra-subtest item selection strategy used in this study was identical to that used by Brown and Weiss (1977) and Gialluca and Weiss (1979) in that items were selected for subsequent administration by selecting the item that provided the highest amount of psychometric information available at the current estimate of θ. Although such maximum information item selection is typically used with maximum likelihood scoring, the present application used a Bayesian scoring method for three reasons:

1. Maximum likelihood scoring requires that an examinee's response vector contain at least one correct and one incorrect response before a θ estimate can be computed;

2. Maximum likelihood scoring does not use prior information as effectively as does Bayesian scoring; and

3. Some response patterns result in a failure of the maximum likelihood procedure to converge upon a θ estimate.

In general, although scores obtained from Bayesian and maximum likelihood methods tend to be highly correlated, they do not yield numerically identical results given the same data. The extent to which these two scoring procedures yield numerically discrepant results was reported by Kingsbury and Weiss (1979). Unfortunately, the issue of the most appropriate choice of scoring algorithm pervades implementations of IRT-based testing in general and hence should be addressed in future research.

In terms of fidelity or correlations with true achievement levels, the adaptive intra-subtest item selection strategy using the .05 termination crite-

rion resulted in lower validity coefficients than the conventionally adminis-
tered subtests. This was likely attributable, in part, to the combined effects
of the Bayesian regressed achievement estimate and the smaller number of items
used for the estimates. This, in turn, caused a restriction in the range of
estimated achievement levels, thus lowering the correlation with true achieve-
ment levels. The same pattern, however, did not appear when the more stringent
.01 termination criterion was used, further supporting the hypothesized effects
of a restricted range of Bayesian achievement level estimates.

Test length reductions of 39% to 67% resulted from the variable termination
criterion of the intra-subtest item selection strategy. In spite of the consid-
erable decrease in test length, there was virtually no loss in the amount of
psychometric information obtained from each subtest.

Inter-Subtest Branching

The use of prior information to determine differential subtest entry into
subsequent subtests appeared to have little effect in terms of number of items
administered over and above that produced by intra-subtest item selection alone.
The extent to which this finding is generalizable across subtests with different
characteristics and different intercorrelations remains to be investigated in
future research.

The addition of inter-subtest branching to the adaptive intra-subtest item
selection generally raised the low validities obtained under the .05 termination
condition using only adaptive intra-subtest item selection to near those ob-
tained with conventional administration. This same effect appeared using the
.01 termination criterion but to a lesser extent.

A possible explanation for the small effects of prior information may lie
in the combination of the use of linear multiple regression estimates to deter-
mine prior ability estimates, coupled with a regressed Bayesian scoring algo-
rithm. That is, the regression estimates themselves tend to underestimate or
regress extreme scores and, together with the tendency of the Bayesian procedure
to further regress the achievement estimates, might tend to offset any benefit
arising from prior information. Perhaps the benefit of inter-subtest branching
is not to further reduce test length but to maintain the original range of
achievement levels against the tendency of Bayesian estimation to regress them.
Maximum likelihood scoring might also be useful at the end of each subtest to
obtain unregressed ability estimates for use in inter-subtest adaptive branch-
ing.

Consistent with the results of test length reduction, mean subtest informa-
tion was not significantly changed when inter-subtest branching was added to the
adaptive intra-subtest item selection strategy. Apparently, the regression-
based achievement estimates had the effect of selecting the same or similar
items as those selected by the testing procedure that did not use differential
subtest entry. This may have been caused by the rather low to moderate inter-
correlations used in the simulation. For example, the range of intercorrela-
tions used for the present study ranged from .14 to .53, with a mean of .34.
Different results might be obtained with more highly correlated tests, such as
those in the Armed Services Vocational Aptitude Battery; ASVAB-9 intercorrela-
tions range from .12 to .75 with a mean of .50.

Conclusions

The results of this study replicated and extended the findings reported by Brown and Weiss (1977) and Gialluca and Weiss (1979). That is, test lengths could be reduced by 39% to 67% using the Brown and Weiss (1977) adaptive testing strategy designed for test batteries.

The design of the present study allowed the separation of the effects due to adaptive intra-subtest item selection from that due to inter-subtest branching. Although the results showed that the intra-subtest item selection accounted for virtually all the reduction in test length, the addition of prior information using inter-subtest branching appeared to have an effect on the observed validities and on mean battery information. Achievement level estimates obtained using inter-subtest branching combined with adaptive intra-subtest item selection showed validities quite comparable to those of conventional tests nearly twice as long. As observed in previous studies, there was a minimal loss of psychometric information due to the adaptive procedure.

Although this study replicated and extended the findings of the previous studies, it, too, was limited by the fact that only a small number of factors were included. The next step should be a large-scale simulation that systematically varies the important sources of variation.

Future research on inter-subtest branching should be concerned with the determination of the conditions under which it is maximally effective. The factors examined should include (1) varying the factorial composition of the subtests as a way of systematically generating realistic correlation matrices; (2) comparing the effects of different scoring algorithms—such as Owen's Bayesian, modal Bayesian (which differs from Owen's in using the mode of the posterior distribution rather than the mean), and maximum likelihood coupled with Bayesian scoring (to eliminate the effects of regressed Bayesian estimates); (3) various subtest orderings; and (4) effects as a function of achievement level as well as averaged over the levels. A double cross-validation design such as used by Gialluca and Weiss (1979) appears to be quite useful in this regard.

Test length in adaptive testing using a variable termination criterion is a function of the magnitude of the termination criterion used. The termination criterion that is optimal for a given situation—the magnitude of information that can be maintained while eliminating non-informative items—also needs to be addressed in future studies. The most important factor in this regard is the information structure of the subtests. Thus, a complete factorial design should also include manipulation of the subtest information curves, based on information curves observed in other test batteries as well as on ideal information curves for use in adaptive testing batteries.

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Table A
Normal Ogive Item Discrimination (a), Difficulty (b), and Pseudo-Guessing (c) Parameter Estimates for the 12 Subtests

Subtest and Item	a	b	c	Subtest and Item	a	b	c	Subtest and Item	a	b	c	Subtest and Item	a	b	c	Subtest and Item	a	b	c
Subtest 1				Subtest 4 (continued)				Subtest 6 (continued)				Subtest 9 (continued)				Subtest 11 (continued)			
1	2.22	1.80	.70	8	---	---	---	14	1.97	-.41	.59	15	1.18	-.56	.31	8	---	---	---
2	1.76	-.06	.53	9	1.52	2.15	.53	15	1.00	2.78	.48	16	.90	2.00	.49	9	.97	2.75	.49
3	2.57	-1.13	.54	10	.97	-.55	.39	16	2.11	3.05	.71	17	1.65	-.82	.39	10	---	---	---
4	.88	.59	.48	11	1.69	-1.13	.48	17	1.24	2.78	.44	18	1.89	.12	.43	11	1.21	1.83	.59
5	3.01	-.52	.30	12	1.94	1.08	.52	18	1.74	2.74	.49	19	1.08	.76	.53	12	2.00	-.05	.30
6	1.29	-.51	.51	13	1.39	.15	.51	Subtest 7				20	1.64	-.58	.37	13	2.36	-.66	.25
7	1.52	1.69	.65	14	---	---	---	1	3.61	-1.29	.46	21	1.23	-.97	.34	14	---	---	---
8	2.14	-.87	.54	15	1.57	-1.37	.52	2	---	---	---	22	1.69	.42	.41	15	.88	.06	.33
9	1.93	.29	.50	16	1.07	.37	.49	3	1.75	1.71	.66	23	2.00	-.97	.42	16	---	---	---
10	1.64	-.04	.47	17	---	---	---	4	1.87	1.42	.64	24	3.50	2.30	.29	17	1.67	.18	.32
Subtest 2				18	2.10	-1.09	.59	5	1.19	.11	.59	Subtest 10				18	---	---	---
1	3.02	1.63	.71	19	1.15	.44	.49	6	1.35	.27	.61	1	1.78	2.30	.34	19	1.37	2.80	.64
2	1.48	-.62	.36	20	1.14	1.95	.39	7	1.67	.08	.63	2	2.04	-.03	.54	20	.75	.44	.35
3	3.62	-1.65	.18	21	1.29	.55	.50	8	1.24	2.38	.47	3	1.23	.93	.61	21	1.12	-.42	.21
4	1.66	.04	.54	22	1.72	-1.34	.53	9	4.30	1.83	0.00	4	2.94	-1.29	.77	22	1.72	-1.00	.43
5	2.44	-.80	.46	Subtest 5				10	1.89	.25	.64	5	---	---	---	23	.73	.49	.32
6	1.28	.24	.53	1	2.13	.62	.49	11	1.23	2.60	.37	6	1.37	.24	.45	24	1.28	2.68	.64
7	2.86	2.94	.86	2	1.20	.76	.31	12	1.84	.91	.62	7	1.32	1.97	.34	25	2.74	.34	.22
8	.90	.58	.50	3	1.05	1.78	.47	13	2.17	2.91	.49	8	1.71	-.20	.57	26	---	---	---
9	2.09	.14	.54	4	.98	.82	.49	14	1.61	2.44	.52	9	1.86	2.45	.21	27	1.66	-.18	.32
10	1.81	.64	.58	5	1.51	-.48	.38	Subtest 8				10	1.13	1.80	.39	28	2.36	-.82	.38
Subtest 3				6	1.42	.43	.64	1	2.00	-.08	.41	11	1.18	2.40	.33	29	1.33	3.00	.62
1	.98	2.55	.46	7	1.25	2.65	.42	2	1.87	1.38	.45	12	1.51	1.48	.39	30	---	---	---
2	---	---	---	8	---	---	---	3	3.12	-.98	.65	13	2.47	2.44	.33	31	.79	.91	.35
3	2.20	-.56	.48	9	1.59	-.71	.41	4	2.61	1.42	.37	14	1.05	1.15	.47	32	.85	2.30	.48
4	2.87	-1.37	.57	10	2.03	1.97	.59	5	3.34	-.95	.53	15	---	---	---	Subtest 12			
5	2.26	-.43	.43	11	.98	1.48	.51	6	2.01	.74	.38	16	1.94	-.68	.64	1	---	---	---
6	1.68	-.73	.51	12	1.09	-.68	.37	7	2.55	.48	.36	17	1.62	1.55	.40	2	3.36	2.09	.10
7	---	---	---	13	1.98	-.64	.47	8	3.41	2.62	.54	18	---	---	---	3	---	---	---
8	---	---	---	14	3.60	2.48	.34	9	.94	2.08	.37	19	2.66	2.12	.18	4	1.29	.75	.55
9	1.35	.48	.61	15	1.36	-1.37	.46	10	.86	1.11	.32	20	1.25	.99	.59	5	1.88	-.32	.54
10	2.14	1.52	.58	16	1.78	-.33	.40	11	1.65	.78	.45	21	1.00	2.51	.37	6	1.68	2.52	.34
11	1.83	-1.28	.52	17	1.05	.75	.50	12	1.06	1.52	.35	22	1.21	1.32	.55	7	---	---	---
12	1.23	.82	.54	18	1.76	2.96	.70	Subtest 9				23	1.92	1.37	.41	8	1.29	3.11	.56
13	1.06	.75	.52	Subtest 6				1	1.39	1.22	.34	24	1.64	.03	.54	9	1.58	-.35	.63
14	2.17	2.50	.66	1	1.55	.18	.66	2	1.00	1.44	.25	25	.88	1.87	.37	10	2.40	-1.01	.90
15	1.51	.01	.50	2	1.30	-.35	.57	3	1.89	1.72	.43	26	---	---	---	11	1.23	2.59	.52
16	1.78	1.58	.70	3	1.42	2.43	.51	4	.77	1.15	.44	27	---	---	---	12	---	---	---
17	1.36	.21	.51	4	1.24	-.02	.55	5	---	---	---	28	---	---	---	13	---	---	---
18	2.57	2.08	.62	5	1.76	-1.07	.53	6	1.44	-.28	.35	29	2.52	2.65	.33	14	1.44	.01	.47
Subtest 4				6	1.62	.53	.55	7	1.11	2.25	.51	Subtest 11				15	2.41	2.23	.27
1	.94	.80	.47	7	1.78	2.61	.67	8	1.29	3.06	.57	1	1.75	-1.36	.61	16	1.67	.53	.57
2	1.40	.49	.51	8	.99	1.07	.40	9	1.04	-.32	.49	2	2.11	1.87	.46	17	2.13	2.99	.61
3	3.36	2.06	.33	9	1.27	1.98	.53	10	1.49	3.09	.64	3	---	---	---	18	.87	2.14	.45
4	1.03	2.59	.30	10	2.74	-1.05	.59	11	---	---	---	4	1.06	.17	.47	19	1.19	2.59	.54
5	1.67	-.20	.52	11	1.85	.68	.70	12	2.06	.47	.44	5	.79	1.34	.43	20	2.01	.44	.55
6	1.92	-.36	.52	12	1.24	2.75	.53	13	1.68	3.16	.59	6	2.42	2.71	.57	21	1.45	1.08	.57
7	2.51	2.26	.42	13	1.66	.82	.55	14	.89	.73	.41	7	1.55	2.44	.50	22	1.68	3.17	.59
																23	1.66	1.39	.59

Note. Dashed lines indicate that an item was rejected in the first phase of the item parameterization procedure.

Table B
Raw Score Regression Weights, Regression Constants, and
Squared Standard Errors of Estimate (SSEE) for Regression
Equations Used to Determine Differential Entry Points,
with Subtests Ordered by Highest R

Subtest	Subtest											
	3	11	5	7	9	4	1	6	2	10	12	8
11	562	-011										
5	394	178	019									
7	177	174	315	043								
9	114	079	266	215	-017							
4	201	123	073	055	249	-041						
1	163	201	130	054	-074	195	-014					
6	085	-068	052	121	300	143	080	-015				
2	011	021	148	178	088	164	106	-008	047			
10	015	035	026	102	143	052	-045	111	222	-006		
12	-051	180	101	038	023	-131	108	-004	167	187	-011	
8	044	008	107	135	013	025	021	072	071	064	121	-001
SSEE	755	740	691	632	668	716	745	764	766	718	772	

Note. Regression constants appear on the main diagonal.

Table C
Raw Score Regression Weights, Regression Constants, and
Squared Standard Errors of Estimate (SSEE) for Regression
Equations Used to Determine Differential Entry Points,
with Subtests Ordered by Number of Items

Subtest	Subtest											
	11	10	9	4	12	6	5	3	7	8	2	1
10	220	006										
9	253	273	-006									
4	225	111	309	-040								
12	241	242	081	-065	002							
6	010	121	335	186	019	-010						
5	168	048	285	120	135	094	022					
3	318	024	049	151	-016	078	223	-004				
7	125	099	152	031	059	094	206	117	046			
8	012	075	014	042	136	072	119	050	146	002		
2	-002	166	039	177	167	-028	117	026	132	067	046	
1	180	-062	-010	178	103	079	096	156	024	019	087	-016
SSEE	924	729	697	765	762	673	538	638	774	712	698	

Note. Regression constants appear on the main diagonal.

Table D
Raw Score Regression Weights, Regression Constants,
and Squared Standard Errors of Estimate (SSEE) for Regression
Equations Used to Determine Differential Entry Points,
with Subtests Ordered Randomly

Subtest	Subtest											
	3	4	11	2	7	6	1	5	8	10	12	9
4	425	-040										
11	482	190	-003									
2	143	263	112	061								
7	219	098	175	222	041							
6	148	233	-029	036	210	-019						
1	170	155	203	105	032	062	-016					
5	226	061	065	137	224	097	079	006				
8	041	018	035	113	152	087	028	128	-003			
10	018	075	042	222	109	-135	-059	044	087	-006		
12	-054	-128	178	158	026	-008	103	092	112	179	-011	
9	046	169	061	036	127	208	-078	195	009	098	017	-006
SSEE	763	726	840	706	810	715	629	786	771	709	547	

Note. Regression constants appear on the main diagonal.

Table E
Mean Information Values (I) at Estimated Achievement Levels ($\hat{\theta}$)
for Subtest 1 under All Testing Conditions

| $\hat{\theta}$ Range | | Conventional Test | | Intra-Subtest Item Selection | | Intra-Subtest Item Selection with Inter-Subtest Branching | | | | | |
| | | | | | | Multiple R | | No. of Items | | Random | |
Lo	Hi	N	I($\hat{\theta}$)	N	I($\hat{\theta}$)	N	I($\hat{\theta}$)	N	I($\hat{\theta}$)	N	I($\hat{\theta}$)
-2.00	-1.80	-	-	-	-	-	-	-	-	-	-
-1.79	-1.60	2	.23	-	-	1	.12	5	.14	3	.13
-1.59	-1.40	11	.58	10	.54	12	.58	8	.55	8	.41
-1.39	-1.20	12	1.35	15	1.34	9	1.49	14	1.49	19	1.41
-1.19	-1.00	22	2.50	15	2.66	17	2.65	15	2.61	16	2.60
-.99	-.80	24	3.79	23	3.91	20	3.60	15	3.71	24	3.84
-.79	-.60	13	4.94	14	5.39	13	5.48	18	5.48	14	5.04
-.59	-.40	11	6.28	27	6.25	18	6.28	18	6.28	16	6.27
-.39	-.20	15	5.78	5	5.58	13	5.49	11	5.57	12	5.78
-.19	.00	27	4.18	27	4.14	16	3.95	22	4.22	25	4.09
.01	.20	40	3.26	26	3.12	32	3.12	31	3.11	21	3.08
.21	.40	8	2.49	-	-	18	2.59	20	2.58	20	2.59
.41	.60	27	2.21	28	2.31	42	2.11	28	2.11	28	2.09
.61	.80	30	1.67	110	1.22	46	1.62	50	1.59	52	1.60
.81	1.00	12	1.41	-	-	29	1.20	29	1.18	32	1.20
1.01	1.20	46	1.00	-	-	9	.87	11	.81	6	.83
1.21	1.40	-	-	-	-	5	.75	3	.72	1	.72
1.41	1.60	-	-	-	-	-	-	2	.78	2	.80
1.61	1.80	-	-	-	-	-	-	-	-	-	-
1.81	2.00	-	-	-	-	-	-	-	-	-	-
Total Group		300	2.75	300	2.73	300	2.71	300	2.73	300	2.71

Table F
Mean Information Values (I) at Estimated Achievement Levels ($\hat{\theta}$)
for Subtest 2 under All Testing Conditions

| $\hat{\theta}$ Range | | Conventional Test | | Intra-Subtest Item Selection | | Intra-Subtest Item Selection with Inter-Subtest Branching | | | | | |
| | | | | | | Multiple R | | No. of Items | | Random | |
Lo	Hi	N	I($\hat{\theta}$)	N	I($\hat{\theta}$)	N	I($\hat{\theta}$)	N	I($\hat{\theta}$)	N	I($\hat{\theta}$)
-2.00	-1.80	7	3.37	-	-	5	3.52	5	3.25	7	2.80
-1.79	-1.60	3	4.44	14	5.28	4	6.70	2	5.62	5	6.08
-1.59	-1.40	-	-	2	6.75	2	6.12	2	6.59	2	6.77
-1.39	-1.20	9	2.47	11	2.13	5	2.94	6	2.82	4	2.63
-1.19	-1.00	21	2.01	25	2.21	9	1.97	10	1.99	4	1.97
-.99	-.80	8	2.20	7	2.55	18	2.18	21	2.19	25	2.16
-.79	-.60	14	2.60	26	2.53	19	2.56	23	2.61	18	2.53
-.59	-.40	29	2.57	43	2.38	32	2.56	18	2.54	27	2.54
-.39	-.20	25	2.40	18	2.52	22	2.39	28	2.38	27	2.38
-.19	.00	37	2.48	37	2.79	25	2.44	28	2.45	22	2.47
.01	.20	17	2.71	13	2.90	35	2.74	30	2.75	21	2.78
.21	.40	23	2.89	24	2.84	24	2.89	28	2.89	28	2.89
.41	.60	28	2.69	-	-	18	2.76	15	2.69	19	2.77
.61	.80	20	2.13	-	-	18	2.32	20	2.33	21	2.28
.81	1.00	-	-	80	1.66	32	1.84	37	1.84	41	1.82
1.01	1.20	59	1.56	-	-	17	1.49	14	1.46	23	1.46
1.21	1.40	-	-	-	-	12	1.16	9	1.16	5	1.16
1.41	1.60	-	-	-	-	3	1.23	4	1.26	1	1.37
1.61	1.80	-	-	-	-	-	-	-	-	-	-
1.81	2.00	-	-	-	-	-	-	-	-	-	-
Total Group		300	2.35	300	2.46	300	2.44	300	2.40	300	2.41

Table G
Mean Information Values (I) at Estimated Achievement Levels (θ̂)
for Subtest 3 under All Testing Conditions

θ̂ Range		Conventional Test		Intra-Subtest Item Selection		Intra-Subtest Item Selection with Inter-Subtest Branching					
						Multiple R		No. of Items		Random	
Lo	Hi	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)
-2.00	-1.80	–	–	–	–	–	–	1	.14	–	–
-1.79	-1.60	4	.78	3	.66	3	.66	2	.92	5	.70
-1.59	-1.40	10	1.86	8	1.76	5	1.58	5	1.69	5	1.37
-1.39	-1.20	3	2.85	12	2.78	10	2.72	9	2.68	15	2.75
-1.19	-1.00	17	2.87	11	2.81	15	2.81	6	2.87	14	2.82
-.99	-.80	23	2.89	13	2.83	17	2.78	28	2.84	16	2.82
-.79	-.60	28	3.46	33	3.35	33	3.30	26	3.31	31	3.35
-.59	-.40	16	4.24	9	4.14	8	4.04	14	4.03	6	4.04
-.39	-.20	10	4.24	27	4.16	34	4.21	20	4.17	34	4.21
-.19	.00	25	3.72	21	3.76	14	3.76	20	3.62	27	3.70
.01	.20	23	3.04	41	2.95	36	2.94	35	3.00	26	3.02
.21	.40	36	2.57	22	2.46	22	2.48	41	2.48	13	2.48
.41	.60	29	2.21	34	2.18	38	2.18	31	2.19	34	2.19
.61	.80	32	1.98	34	1.90	35	1.90	29	1.88	34	1.88
.81	1.00	15	1.81	–	–	–	–	13	1.70	0	–
1.01	1.20	13	1.75	–	–	–	–	9	1.67	–	–
1.21	1.40	11	1.84	32	1.78	30	1.76	5	1.82	40	1.78
1.41	1.60	5	2.13	–	–	–	–	5	2.04	–	–
1.61	1.80	–	–	–	–	–	–	1	2.35	–	–
1.81	2.00	–	–	–	–	–	–	–	–	–	–
Total Group		300	2.72	300	2.76	300	2.76	300	2.72	300	2.76

Table H
Mean Information Values (I) at Estimated Achievement Levels (θ̂)
for Subtest 4 under All Testing Conditions

θ̂ Range		Conventional Test		Intra-Subtest Item Selection		Intra-Subtest Item Selection with Inter-Subtest Branching					
						Multiple R		No. of Items		Random	
Lo	Hi	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)
-2.00	-1.80	1	.62	–	–	1	.44	–	–	–	–
-1.79	-1.60	–	–	1	.80	2	.76	5	.96	1	.86
-1.59	-1.40	5	1.70	12	1.62	7	1.83	5	1.70	7	1.80
-1.39	-1.20	11	2.51	11	2.52	11	2.50	14	2.52	11	2.58
-1.19	-1.00	19	3.19	13	3.12	9	3.14	14	3.08	16	3.13
-.99	-.80	13	3.37	16	3.31	13	3.30	20	3.31	14	3.30
-.79	-.60	22	3.30	24	3.22	30	3.20	20	3.20	24	3.20
-.59	-.40	33	3.28	33	3.23	31	3.23	40	3.24	32	3.22
-.39	-.20	32	3.41	14	3.42	33	3.42	17	3.42	25	3.42
-.19	.00	34	3.52	35	3.52	25	3.52	29	3.52	30	3.52
.01	.20	28	3.51	35	3.45	26	3.47	34	3.48	39	3.48
.21	.40	16	3.41	28	3.31	34	3.30	34	3.30	23	3.29
.41	.60	17	3.30	3	3.19	14	3.18	19	3.20	17	3.18
.61	.80	17	3.20	34	3.10	19	3.09	13	3.11	18	3.09
.81	1.00	17	3.09	1	3.09	10	3.04	13	3.03	11	3.03
1.01	1.20	8	2.94	17	2.83	2	2.92	13	3.85	10	3.88
1.21	1.40	12	2.71	11	2.66	22	2.64	14	2.62	20	2.61
1.41	1.60	5	2.40	4	2.38	4	2.37	3	2.30	1	2.32
1.61	1.80	6	2.90	2	2.67	–	–	–	–	–	–
1.81	2.00	4	5.46	6	5.03	1	7.07	3	6.24	1	5.54
Total Group		300	3.25	300	3.17	300	3.15	300	3.16	300	3.17

Table I
Mean Information Values (I) at Estimated Achievement Levels (θ̂)
for Subtest 5 under All Testing Conditions

θ Range		Conventional Test		Intra-Subtest Item Selection		Intra-Subtest Item Selection with Inter-Subtest Branching					
						Multiple R		No. of Items		Random	
Lo	Hi	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)
-2.00	-1.80	–	–	–	–	–	–	–	–	–	–
-1.79	-1.60	–	–	–	–	–	–	2	.50	2	.36
-1.59	-1.40	2	.85	6	.70	3	.46	10	.71	3	.66
-1.39	-1.20	11	1.18	14	1.13	9	.71	14	1.16	16	1.13
-1.19	-1.00	20	1.75	29	1.79	9	1.12	12	1.79	13	1.79
-.99	-.80	18	2.59	5	2.80	9	1.77	22	2.61	23	2.56
-.79	-.60	33	3.61	31	3.47	20	2.57	14	3.44	21	3.36
-.59	-.40	18	4.23	13	4.16	18	3.43	21	4.09	26	4.09
-.39	-.20	30	4.26	45	4.17	25	4.08	32	4.18	24	4.17
-.19	.00	25	3.85	14	3.90	31	4.17	25	3.88	12	3.78
.01	.20	27	3.41	21	3.40	19	3.87	27	3.32	30	3.36
.21	.40	25	3.19	38	3.14	33	3.42	19	3.14	20	3.14
.41	.60	27	3.30	11	3.23	29	3.14	25	3.21	29	3.20
.61	.80	16	3.39	23	3.35	20	3.22	26	3.34	30	3.34
.81	1.00	18	3.18	20	3.19	17	3.34	16	3.05	16	3.05
1.01	1.20	13	2.74	16	2.58	19	3.06	16	2.61	12	2.58
1.21	1.40	6	2.26	5	2.38	6	2.25	12	2.23	10	2.15
1.41	1.60	9	1.98	5	1.96	10	1.91	3	1.84	10	1.86
1.61	1.80	–	–	–	–	3	1.84	4	1.90	1	1.86
1.81	2.00	2	5.80	4	1.96	1	1.98	–	–	2	1.97
Total	Group	300	3.22	300	3.09	300	3.08	300	3.05	300	3.03

Table J
Mean Information Values (I) at Estimated Achievement Levels (θ̂)
for Subtest 6 under All Testing Conditions

θ Range		Conventional Test		Intra-Subtest Item Selection		Intra-Subtest Item Selection with Inter-Subtest Branching					
						Multiple R		No. of Items		Random	
Lo	Hi	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)
-2.00	-1.80	–	–	–	–	1	.04	–	–	–	–
-1.79	-1.60	1	.25	–	–	4	.14	2	.15	3	.14
-1.59	-1.40	5	.43	9	.32	4	.42	7	.46	4	.31
-1.39	-1.20	12	1.23	19	1.03	10	.99	11	1.10	9	1.12
-1.19	-1.00	20	2.07	7	2.18	13	2.05	13	1.93	12	2.08
-.99	-.80	10	2.59	24	2.56	14	2.58	9	2.55	13	2.56
-.79	-.60	18	2.43	9	2.38	29	2.40	23	2.40	21	2.46
-.59	-.40	25	2.23	26	2.18	19	2.20	26	2.18	16	2.18
-.39	-.20	32	2.16	18	2.12	34	2.10	32	2.10	48	2.10
-.19	.00	23	2.13	49	2.10	38	2.07	45	2.08	35	2.07
.01	.20	36	2.14	19	2.13	17	2.12	16	1.12	15	2.12
.21	.40	35	2.29	16	2.28	28	2.25	23	2.26	30	2.26
.41	.60	16	2.49	41	2.48	21	2.42	20	2.43	22	2.41
.61	.80	9	2.66	–	–	10	2.60	18	2.60	13	2.60
.81	1.00	14	2.60	18	2.56	21	2.52	17	2.52	26	2.52
1.01	1.20	26	2.28	–	–	9	2.27	15	2.24	11	2.34
1.21	1.40	12	1.86	45	1.77	17	1.82	15	1.81	17	1.83
1.41	1.60	5	1.56	–	–	8	1.41	5	1.45	4	1.47
1.61	1.80	–	–	–	–	3	1.17	3	1.18	1	1.24
1.81	2.00	1	1.86	–	–	–	–	–	–	–	–
Total	Group	300	2.17	300	2.08	300	2.10	300	2.10	300	2.15

Table K
Mean Information Values (I) at Estimated Achievement Levels ($\hat\theta$)
for Subtest 7 under All Testing Conditions

| $\hat\theta$ Range | | Conventional Test | | Intra-Subtest Item Selection | | Intra-Subtest Item Selection with Inter-Subtest Branching | | | | | |
| | | | | | | Multiple R | | No. of Items | | Random | |
Lo	Hi	N	I($\hat\theta$)	N	I($\hat\theta$)	N	I($\hat\theta$)	N	I($\hat\theta$)	N	I($\hat\theta$)
-2.00	-1.80	–	–	–	–	–	–	–	–	1	.06
-1.79	-1.60	17	.31	–	–	5	.18	6	.27	7	.23
-1.59	-1.40	–	–	16	.69	3	1.28	10	1.13	6	1.11
-1.39	-1.20	–	–	–	–	2	3.07	–	–	1	3.26
-1.19	-1.00	–	–	–	–	–	–	1	3.04	–	–
-.99	-.80	6	1.47	–	–	3	1.16	10	1.04	6	1.30
-.79	-.60	22	.73	41	.56	23	.58	24	.56	20	.57
-.59	-.40	40	.45	14	.45	27	.43	34	.42	34	.43
-.39	-.20	31	.65	89	.66	41	.58	35	.60	42	.61
-.19	.00	48	1.00	–	–	38	1.00	26	.96	31	.96
.01	.20	21	1.47	25	1.42	39	1.47	37	1.38	35	1.38
.21	.40	23	1.77	30	1.87	23	1.79	18	1.82	18	1.81
.41	.60	43	1.92	82	1.90	31	1.89	30	1.89	39	1.89
.61	.80	38	1.88	–	–	34	1.83	38	1.83	30	1.83
.81	1.00	6	1.84	–	–	26	1.79	23	1.80	20	1.79
1.01	1.20	–	–	–	–	2	1.87	4	1.96	1	2.12
1.21	1.40	–	–	–	–	–	–	–	–	1	2.26
1.41	1.60	–	–	–	–	–	–	–	–	–	–
1.61	1.80	–	–	–	–	–	–	–	–	–	–
1.81	2.00	5	4.54	3	3.68	3	7.14	4	6.49	8	9.10
Total Group		300	1.25	300	1.19	300	1.31	300	1.28	300	1.41

Table L
Mean Information Values (I) at Estimated Achievement Levels ($\hat\theta$)
for Subtest 8 under All Testing Conditions

| $\hat\theta$ Range | | Conventional Test | | Intra-Subtest Item Selection | | Intra-Subtest Item Selection with Inter-Subtest Branching | | | | | |
| | | | | | | Multiple R | | No. of Items | | Random | |
Lo	Hi	N	I($\hat\theta$)	N	I($\hat\theta$)	N	I($\hat\theta$)	N	I($\hat\theta$)	N	I($\hat\theta$)
-2.00	-1.80	–	–	–	–	–	–	–	–	–	–
-1.79	-1.60	–	–	–	–	1	.20	–	–	1	.03
-1.59	-1.40	6	.09	6	.08	6	.06	5	.09	10	.10
-1.39	-1.20	29	.73	19	.65	7	.66	17	.59	13	.48
-1.19	-1.00	14	2.06	5	2.23	16	2.29	9	2.60	7	2.78
-.99	-.80	–	–	1	3.82	1	3.88	5	3.97	3	4.06
-.79	-.60	22	3.38	–	–	14	3.33	18	3.24	7	3.44
-.59	-.40	24	2.30	49	2.18	52	1.98	37	1.96	41	1.93
-.39	-.20	36	1.54	49	1.48	30	1.46	37	1.46	35	1.46
-.19	.00	41	1.76	20	1.67	33	1.66	34	1.68	40	1.66
.01	.20	22	2.35	41	2.18	31	2.29	28	2.30	24	2.22
.21	.40	24	3.43	25	2.98	29	3.24	21	3.27	32	3.16
.41	.60	19	4.52	30	4.36	20	4.48	23	4.48	20	4.46
.61	.80	12	4.97	12	4.92	16	4.91	15	4.92	17	4.92
.81	1.00	8	4.70	4	4.83	10	4.75	17	4.77	12	4.75
1.01	1.20	14	4.65	10	4.74	9	4.69	14	4.66	11	4.66
1.21	1.40	6	5.25	2	4.92	6	5.10	5	5.06	4	5.14
1.41	1.60	15	5.31	6	5.33	5	5.26	3	5.14	10	5.25
1.61	1.80	2	4.55	10	4.68	4	4.82	2	4.84	3	4.57
1.81	2.00	8	2.96	11	2.11	10	2.61	10	2.62	10	2.65
Total Group		300	2.75	300	2.58	300	2.73	300	2.76	300	2.70

Table M
Mean Information Values (I) at Estimated Achievement Levels (θ̂)
for Subtest 9 under All Testing Conditions

| θ̂ Range | | Conventional Test | | Intra-Subtest Item Selection | | Intra-Subtest Item Selection with Inter-Subtest Branching | | | | | |
| Lo | Hi | | | | | Multiple R | | No. of Items | | Random | |
		N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)
-2.00	-1.80	-	-	-	-	-	-	-	-	-	-
-1.79	-1.60	-	-	-	-	3	.29	-	-	3	.41
-1.59	-1.40	3	.85	4	.80	5	.94	6	.86	6	.92
-1.39	-1.20	13	1.85	13	1.56	6	1.71	17	1.71	9	1.38
-1.19	-1.00	24	2.89	18	2.83	17	2.94	13	2.77	9	2.83
-.99	-.80	27	3.94	15	3.87	16	3.84	25	3.88	19	3.93
-.79	-.60	16	4.55	27	4.49	17	4.50	16	4.49	17	4.48
-.59	-.40	21	4.79	28	4.70	30	4.60	21	4.68	29	4.69
-.39	-.20	28	4.75	28	4.66	33	4.66	28	4.67	31	4.66
-.19	.00	29	4.78	26	4.69	29	4.68	37	4.69	24	4.68
.01	.20	27	4.95	25	4.86	33	4.94	31	4.92	31	4.90
.21	.40	22	5.23	23	5.23	23	5.25	20	5.23	30	5.23
.41	.60	13	5.24	24	5.24	22	5.19	27	5.19	26	5.22
.61	.80	23	4.89	10	4.76	20	4.74	14	4.70	20	4.68
.81	1.00	16	4.16	20	4.13	17	4.18	12	4.14	16	4.18
1.01	1.20	12	3.66	13	3.45	10	3.54	14	3.57	11	3.49
1.21	1.40	12	3.39	9	3.33	6	3.28	7	3.31	10	3.30
1.41	1.60	4	3.32	10	3.22	8	3.20	8	3.22	7	3.22
1.61	1.80	4	3.34	4	3.21	2	3.11	1	3.18	-	-
1.81	2.00	6	3.76	3	4.85	3	5.01	3	5.52	2	4.84
Total Group		300	4.26	300	4.23	300	4.33	300	4.24	300	4.32

Table N
Mean Information Values (I) at Estimated Achievement Levels (θ̂)
for Subtest 10 under All Testing Conditions

| θ̂ Range | | Conventional Test | | Intra-Subtest Item Selection | | Intra-Subtest Item Selection with Inter-Subtest Branching | | | | | |
| Lo | Hi | | | | | Multiple R | | No. of Items | | Random | |
		N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)	N	I(θ̂)
-2.00	-1.80	-	-	-	-	-	-	-	-	-	-
-1.79	-1.60	-	-	-	-	1	.14	2	.13	-	-
-1.59	-1.40	7	.30	6	.47	1	.60	2	.30	4	.47
-1.39	-1.20	1	1.02	-	-	4	.82	-	-	9	.75
-1.19	-1.00	14	1.11	18	1.07	10	1.06	8	1.06	12	1.06
-.99	-.80	27	1.09	17	1.03	16	1.03	35	1.04	13	1.03
-.79	-.60	20	1.20	17	1.11	26	1.19	28	1.23	29	1.18
-.59	-.40	22	1.57	29	1.44	34	1.57	21	1.50	29	1.57
-.39	-.20	30	2.18	31	2.09	29	2.06	34	2.10	28	2.10
-.19	.00	35	2.74	72	2.64	34	2.65	38	2.57	35	2.65
.01	.20	31	3.08	3	2.97	36	2.94	23	2.94	23	2.95
.21	.40	31	2.99	32	2.89	25	2.88	27	2.88	24	2.87
.41	.60	23	2.77	28	2.59	27	2.67	33	2.68	31	2.63
.61	.80	21	2.66	11	2.57	16	2.54	12	2.54	19	2.54
.81	1.00	17	2.89	12	2.78	8	2.80	13	2.83	13	2.79
1.01	1.20	7	3.47	8	3.64	12	3.44	8	3.57	11	3.55
1.21	1.40	3	4.43	5	4.54	8	4.37	8	4.42	9	4.40
1.41	1.60	7	5.59	3	5.16	4	5.54	3	5.50	4	5.31
1.61	1.80	-	-	5	6.47	3	6.77	2	6.58	4	6.27
1.81	2.00	4	9.23	3	9.58	6	10.52	3	8.17	3	10.74
Total Group		300	2.46	300	2.40	300	2.53	300	2.33	300	2.42

Table O
Mean Information Values (I) at Estimated Achievement Levels ($\hat{\theta}$)
for Subtest 11 under All Testing Conditions

$\hat{\theta}$ Range		Conventional Test		Intra-Subtest Item Selection		Intra-Subtest Item Selection with Inter-Subtest Branching					
						Multiple R		No. of Items		Random	
Lo	Hi	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$
-2.00	-1.80	–	–	–	–	–	–	–	–	3	.20
-1.79	-1.60	4	.60	2	.54	5	.56	–	–	5	.84
-1.59	-1.40	8	1.12	8	.93	1	.91	16	.94	5	.84
-1.39	-1.20	13	1.96	19	1.70	1	1.76	12	1.72	14	1.85
-1.19	-1.00	21	3.33	14	3.36	1	3.14	21	3.41	12	3.06
-.99	-.80	22	5.12	24	5.21	2	5.09	22	5.19	23	5.16
-.79	-.60	25	6.68	24	6.81	1	6.78	26	6.85	22	6.90
-.59	-.40	21	7.26	29	7.18	2	7.18	31	7.18	35	7.19
-.39	-.20	17	7.06	20	7.03	2	7.04	20	7.04	24	7.04
-.19	.00	32	7.20	24	7.20	2	7.17	24	7.22	26	7.19
.01	.20	28	7.93	30	8.08	2	7.91	21	8.01	19	7.91
.21	.40	22	8.58	21	8.54	1	8.53	11	8.53	18	8.58
.41	.60	14	7.86	21	7.45	1	7.35	18	7.61	23	7.56
.61	.80	11	5.36	10	5.49	1	5.46	17	5.41	17	5.40
.81	1.00	23	3.70	11	3.27	2	3.40	16	3.22	16	3.60
1.01	1.20	20	2.73	15	2.49	1	2.58	15	2.53	17	2.55
1.21	1.40	10	2.15	5	2.13	1	2.10	6	2.09	9	2.08
1.41	1.60	5	2.09	16	1.99	1	2.00	12	1.99	7	2.00
1.61	1.80	2	2.52	2	2.33		2.32	5	2.32	3	2.46
1.81	2.00	2	2.92	5	2.72		2.77	7	2.72	2	2.95
Total Group		300	5.50	300	5.55	30	5.29	300	5.28	300	5.55

Table P
Mean Information Values (I) at Estimated Achievement Levels ($\hat{\theta}$)
for Subtest 12 under All Testing Conditions

$\hat{\theta}$ Range		Conventional Test		Intra-Subtest Item Selection		Intra-Subtest Item Selection with Inter-Subtest Branching					
						Multiple R		No. of Items		Random	
Lo	Hi	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$	N	$I(\hat{\theta})$
-2.00	-1.80	–	–	–	–	–	–	–	–	–	–
-1.79	-1.60	–	–	–	–	–	–	–	–	–	–
-1.59	-1.40	1	.04	–	–	–	–	1	.04	1	.03
-1.39	-1.20	7	.11	2	.11	2	.13	3	.15	3	.12
-1.19	-1.00	13	.29	11	.28	11	.27	12	.29	9	.26
-.99	-.80	31	.53	26	.52	20	.46	23	.51	18	.54
-.79	-.60	33	.86	26	.96	26	.84	26	.89	32	.86
-.59	-.40	26	1.25	8	1.39	41	1.25	39	1.26	37	1.25
-.39	-.20	21	1.77	82	1.58	30	1.69	34	1.66	30	1.69
-.19	.00	34	2.09	11	1.99	32	2.04	30	2.03	25	2.03
.01	.20	36	2.31	46	2.28	35	2.29	31	2.32	29	2.32
.21	.40	21	2.60	13	2.71	19	2.55	22	2.62	19	2.60
.41	.60	23	2.84	23	2.79	31	2.79	19	2.81	28	2.79
.61	.80	8	2.82	15	2.82	17	2.77	6	2.76	14	2.78
.81	1.00	12	2.63	–	–	20	2.51	24	2.55	27	2.53
1.01	1.20	17	2.32	36	2.37	7	2.20	17	2.29	14	2.30
1.21	1.40	11	2.13	–	–	8	2.04	8	2.04	10	2.02
1.41	1.60	4	2.16	–	–	–	–	1	2.13	1	2.10
1.61	1.80	2	2.88	–	–	–	–	1	2.70	1	2.92
1.81	2.00	–	–	1	6.52	1	7.86	3	7.69	1	6.30
Total Group		300	1.75	300	1.81	300	1.84	300	1.83	300	1.85